Eur. Phys. J. C 39, 435–438 (2005)
Digital Object Identifier (DOI) 10.1140/epjc/s2004-02102-2

THE EUROPEAN
PHYSICAL JOURNAL C

Energy dependence of gap survival probability and antishadowing

S.M. Troshin[a], N.E. Tyurin

Institute for High Energy Physics, Protvino, Moscow Region, 142281 Russia

Received: 8 April 2004 / Revised version: 31 October 2004 /
Published online: 25 January 2005 –

Abstract. We discuss the energy dependence of the gap survival probability which follows from the rational form of the amplitude unitarization. In contrast to the eikonal form of unitarization which leads to a decreasing energy dependence of the gap survival probability, we predict a non-monotonous form for this dependence.

Studies of the processes with large rapidity gaps are very important as a tool for the search of new physics. For the first time such processes have been discussed in [1–3]. The predictive power of QCD calculations for cross-sections of such processes is affected by the uncertainties related to the soft interactions (rescattering) in initial and final states. The dynamics of such interactions is accounted for by the introduction of a factor which is known as the gap survival probability [3], i.e. the probability to keep away inelastic interactions which can result in filling up by hadrons of the large rapidity gaps. The energy dependence and magnitude of the gap survival probability is an important issue e.g. in the studies of Higgs production in double diffractive exclusive and inclusive processes at Tevatron and the LHC (cf. [4]). Extensive studies of the magnitude and energy dependence of the gap survival probability have been performed, and the results of these studies can be found e.g. in [5–8].

The gap survival probability $\langle |S|^2 \rangle$ is determined by the relation [3]

$$\langle |S|^2 \rangle = \frac{\int_0^\infty D_{\mathrm{H}}(b)|S(s,b)|^2 \mathrm{d}^2 b}{\int_0^\infty D_{\mathrm{H}}(b) \mathrm{d}^2 b}\,, \qquad (1)$$

where $D_{\mathrm{H}}(b)$ is the probability to observe a specific hard interaction in the collision of the hadrons h_1 and h_2, and $P(s,b) \equiv |S(s,b)|^2$, where S is the elastic scattering S-matrix, i.e. P is the probability of the absence of the inelastic interactions. In the eikonal formalism which is usually used for an estimation of $\langle |S|^2 \rangle$, the probability obeys $P(s,b) = \exp(-\Omega(s,b))$. All estimations of the gap survival probability performed on the basis of the eikonal amplitude unitarization lead to a decreasing energy dependence of this quantity. Therefore rather small values of the cross-sections for diffractive Higgs productions are expected at the LHC energies [9,10].

However, there is an alternative approach to unitarization which utilizes a rational representation and leads, as it will be shown below, to a non-monotonous energy dependence of the gap survival probability. Arguments based on the analytical properties of the scattering amplitude [11] provide support for the rational form of unitarization. In potential scattering the rational form of the unitarization corresponds to an approximate wave function which changes both the phase and amplitude of the wave. The rational form of unitarization in quantum field theory is based on the relativistic generalization [12] of the Heitler equation [13]. In the U-matrix approach based on the rational form of unitarization, the elastic scattering amplitude in the impact parameter representation has the form

$$f(s,b) = \frac{U(s,b)}{1 - \mathrm{i}U(s,b)}\,, \qquad (2)$$

where $U(s,b)$ is the generalized reaction matrix, which is considered to be an input dynamical quantity similar to the eikonal function. The unitarity equation for the elastic amplitude $f(s,b)$ rewritten at high energies has the form

$$\mathrm{Im} f(s,b) = |f(s,b)|^2 + \eta(s,b)\,, \qquad (3)$$

where the inelastic overlap function

$$\eta(s,b) \equiv \frac{1}{4\pi} \frac{\mathrm{d}\sigma_{\mathrm{inel}}}{\mathrm{d}b^2}$$

is the sum of all inelastic channel contributions. The inelastic overlap function is related to $U(s,b)$ according to (2) and (3) as follows:

$$\eta(s,b) = \frac{\mathrm{Im}U(s,b)}{|1 - \mathrm{i}U(s,b)|^2}\,. \qquad (4)$$

The probability is

$$P(s,b) \equiv |S(s,b)|^2 = \left| \frac{1 + \mathrm{i}U(s,b)}{1 - \mathrm{i}U(s,b)} \right|^2 . \qquad (5)$$

Unitarity of the scattering matrix implies, in principle, the existence at high enough energies $s > s_0$, where s_0 is

[a] e-mail: Sergey.Troshin@ihep.ru

the threshold of the new scattering mode, the antishadow one. It has been revealed in [14], and the effects related to antishadowing at the LHC energies have been discussed in [15]. The most important feature of this mode is the self-damping of the contribution from the inelastic channels. The rational form of unitarization provides a smooth transition beyond the black disk limit, where the antishadow scattering mode is realized, i.e. at high energies and at small impact parameters the elastic scattering channel can play a dominating role.

There are experimental indications that this mode can indeed occur at very high energies. The analysis of the experimental data on high-energy diffractive scattering shows that the effective interaction area expands with energy and the interaction intensity–opacity increases with energy at fixed impact parameter. At the Tevatron highest energy, $\sqrt{s} = 1800\,\mathrm{GeV}$, the elastic scattering amplitude is very close to the black disk limit at $b = 0$ [16], i.e.

$$\mathrm{Im} f(s, b = 0) = 0.492 \pm 0.008\,.$$

The central inelastic collisions of the hadrons are far from an amalgamation of the two hadrons in one region of space as it was shown in [17], and the persistence of longitudinal momentum takes place at very high energies.

The rational form of amplitude unitarization can be put into agreement with the experimental data using various model parameterizations for the U-matrix. They all lead to the same qualitative predictions, which reflect general properties of this unitarization scheme. Originally the Regge-pole model was used to get an explicit form of the U-matrix [18] and a good description of the experimental data has been obtained in this model [19] as well as in its dipole–pomeron modification [20].

We use the model for the U-matrix based on the ideas of chiral quark approaches [21]. It is in a good agreement with the data [15,22,23] and is also applicable to the large angle scattering. We would like to stress here that the qualitative conclusions of the present paper do not depend on the particular U-matrix parameterization.

In this model the picture of a hadron consisting of constituent quarks embedded in a quark condensate is used. This picture implies that overlapping and interaction of peripheral clouds occurs at the first stage of the hadron interaction. Non-linear field couplings could transform then the kinetic energy to internal energy and as a result massive virtual quarks appear in the overlapping region. These quarks generate an effective field. Valence constituent quarks located in the central part of hadrons are supposed to scatter simultaneously in a quasi-independent way in this field. Massive virtual quarks play the role of scatterers for the valence quarks in elastic scattering, and their hadronization leads to a soft production process of secondary particles in the central region [24]. The number of such scatterers was estimated to be

$$\tilde{N}(s, b) \propto \frac{(1 - \langle k_Q \rangle)\sqrt{s}}{m_Q} D_{\mathrm{C}}(b)\,, \tag{6}$$

under the assumption that part of the hadron energy carried by the outer condensate clouds is released in the overlap region to generate massive quarks, where m_Q is a constituent quark mass and $\langle k_Q \rangle$ the average fraction of hadron energy carried by the constituent valence quarks. The function $D_{\mathrm{C}}(b)$ is a convolution of the two condensate distributions $D_{\mathrm{c}}^{h_1}(b)$ and $D_{\mathrm{c}}^{h_2}(b)$ inside the hadron h_1 and h_2.

We will consider for simplicity the case of a pure imaginary amplitude, i.e. $U \to iu$. The function $u(s, b)$ is represented in the model as a product of the averaged quark amplitudes $\langle f_Q \rangle$,

$$u(s, b) = \prod_{i=1}^{N} \langle f_{Q_i}(s, b) \rangle\,, \tag{7}$$

in accordance with the assumed quasi-independent nature of the valence quark scattering, and N is the total number of valence quarks in the colliding hadrons. The essential point here is the rise with energy of the number of scatterers like $\sqrt{s}$. The b-dependence of the function $\langle f_Q \rangle$ has the simple form $\langle f_Q(b) \rangle \propto \exp(-m_Q b/\xi)$. The generalized reaction matrix gets the following form:

$$u(s, b) = g \left(1 + \alpha \frac{\sqrt{s}}{m_Q}\right)^N \exp\left(-\frac{Mb}{\xi}\right)\,, \tag{8}$$

where $M = \sum_{Q=1}^{N} m_Q$. Here m_Q is the mass of the constituent quark, which is taken to be $0.35\,\mathrm{GeV}$[1].

This model provides a linear dependence on $\sqrt{s}$ for the total cross-sections, i.e. $\sigma_{\mathrm{tot}} = a + c\sqrt{s}$ in the limited energy range $\sqrt{s} \le 0.5\,\mathrm{TeV}$. Such a behavior and model predictions for higher energies are, as already mentioned, in agreement (Fig. 1) with the experimental data on total, elastic and diffractive scattering cross-sections [22,23].

This unitarization approach leads to the following asymptotical dependences: $\sigma_{\mathrm{tot}} \propto \ln^2 s$ and $\sigma_{\mathrm{inel}} \propto \ln s$, which are the same for the various models and reflect the essential properties of this unitarization scheme.

Thus, now the probability $P(s, b) = |S(s, b)|^2$ can be calculated in a straightforward way, i.e. we use for the function $u(s, b)$ formula (8), with parameter fixed from the total cross-section fit, and the relation of the $S(s, b)$- and the U-matrix (5). The impact parameter dependence of $P(s, b)$ for the different energies is presented on Fig. 2.

To calculate the gap survival probability $\langle |S|^2 \rangle$ we need to know the probability of the hard interactions $D_{\mathrm{H}}(b)$. To be specific we consider the hard central production processes

$$p + p \to p + \mathrm{gap} + (\mathrm{Higgs\ or\ } jj) + \mathrm{gap} + p\,. \tag{9}$$

The interest in such processes is related to the clear experimental signature and the significant signal-to-background ratio.

We can write down the probability of the hard interactions in the model as a convolution

$$D_{\mathrm{H}}(b) = \sigma_{\mathrm{H}} \int D_{\mathrm{c}}^{h_1}(\mathbf{b}_1) w_{\mathrm{H}}(\mathbf{b} + \mathbf{b}_1 - \mathbf{b}_2) D_{\mathrm{c}}^{h_2}(\mathbf{b}_2) \mathrm{d}\mathbf{b}_1 \mathrm{d}\mathbf{b}_2\,, \tag{10}$$

[1] The other parameters have the following values: $g = 0.24$, $\xi = 2.5$, $\alpha = 0.56 \cdot 10^{-4}$; these have been obtained from the fit to the total hp cross-sections [22].

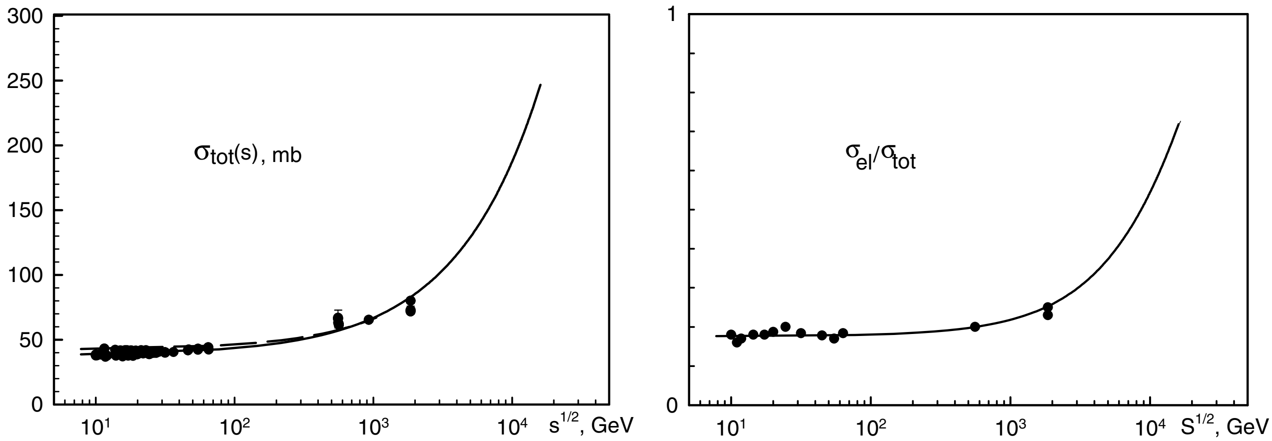

Fig. 1. Total and ratio of elastic to total cross-sections of pp and $\bar{p}p$ interactions

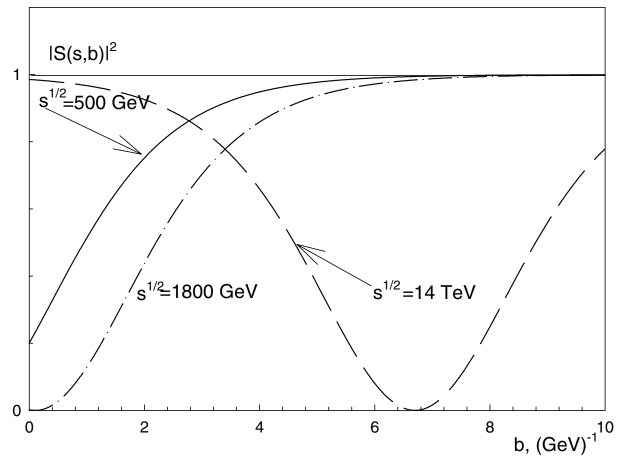

Fig. 2. Impact parameter dependence of the probability $P(s,b) = |S(s,b)|^2$ at the three values of energy $\sqrt{s} = 500\,\mathrm{GeV}$ (shadow scattering mode), $\sqrt{s} = 1800\,\mathrm{GeV}$ (black disk limit at $b = 0$) and $\sqrt{s} = 14\,\mathrm{TeV}$ (shadow and antishadow scattering modes)

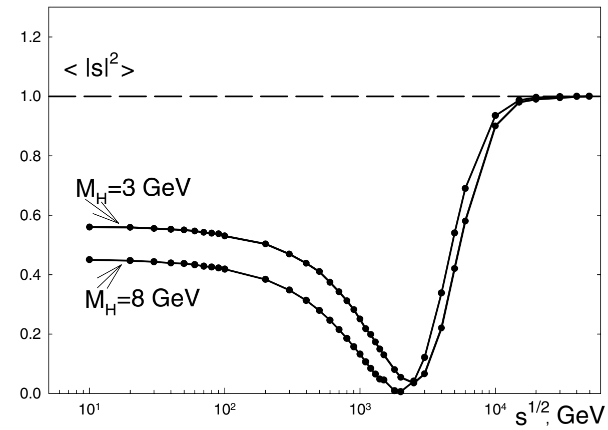

Fig. 3. Energy dependence of the gap survival probability $\langle|S|^2\rangle$

where $w_{\mathrm{H}}(\mathbf{b}+\mathbf{b}_1-\mathbf{b}_2)$ is the probability of hard condensate (parton) interactions. It is natural to assume that the probability w_{H} has a much steeper impact parameter dependence than the functions $D_{\mathrm{c}}^{h_i}(b)$, and, therefore, the impact parameter dependence of w_{H} determines the behavior of $D_{\mathrm{H}}(b)$. Thus, we assume a simple exponential dependence for the function $D_{\mathrm{H}}(b)$, i.e.

$$D_{\mathrm{H}}(b) \simeq \sigma_{\mathrm{H}} \exp(-M_{\mathrm{H}} b)\,, \qquad (11)$$

where the mass M_{H} is determined by the hard scale of the process. We perform numerical calculations of the gap survival probability $\langle|S|^2\rangle$ using (1), (5) and (11).

We take the hard scale M_{H} to be determined by the mass of the J/Ψ-meson, i.e. $M_{\mathrm{H}} = M_{J/\Psi}$. This choice is in accord with the fact that the production of the J/Ψ-meson can be treated as a hard process and therefore its mass sets a hard scale [25]. Lower values of M_{H} lie in the soft region[2].

[2] In (11) σ_{H} can be interpreted as the probability of hot spot formation under condensate interaction, and $R_{\mathrm{H}} \simeq 1/M_{\mathrm{H}}$ is the radius of this hot spot.

It should be noted that numerical results are rather stable and depend weakly on the scale M_{H}, when it is in the hard region, i.e. $M_{\mathrm{H}} \geq M_{J/\Psi}$; e.g. for illustration we used the value $M_{\mathrm{H}} = 8\,\mathrm{GeV}$ and this leads to slightly lower values of the gap survival probability at low energies. Results of the calculations are presented in Fig. 3. One can notice that the gap survival probability reaches its minimal values at the Tevatron highest energy. This is not surprising since the scattering at this energy is very close to the black disk limit.

The asymptotical behavior of the gap survival probability has the form

$$\langle|S|^2\rangle \simeq 1 - \frac{\xi M_{\mathrm{H}}}{2m_Q} s^{-\frac{\xi M_{\mathrm{H}}}{2m_Q}} \ln s\,. \qquad (12)$$

The two unitarization schemes (U-matrix and eikonal) lead to different predictions for the gap survival probability in the limit $s \to \infty$; eikonal unitarization predicts $\langle|S|^2\rangle = 0$ at $s \to \infty$, while the U-matrix formalism gives $\langle|S|^2\rangle = 1$. The latter is a result of the transition to the antishadow scattering mode in the U-matrix unitarization [14], when the amplitude becomes $|f(s,b)| > 1/2$ (in the case of imaginary eikonal the scattering amplitude never exceeds the black disk limit $|f(s,b)| \leq 1/2$). It should be noted that the Froissart–Martin bound implies the unitarity (not black disk) limit for the partial amplitudes. When the amplitude exceeds the black disk limit (in central collisions at high energies), then the scattering at such impact parameters turns out to be of an antishadow nature. In this antishadow scattering mode the elastic amplitude increases with decrease of the inelastic channels contribution at small impact parameters and most of inelastic interactions occur in the peripheral region; the inelastic overlap function has a peripheral impact parameter profile, which is the main reason of the large gap survival probability.

Numerical predictions for the gap survival probability obtained here depend on the particular parameterization for the U-matrix, but the qualitative picture of the energy behavior of $\langle|S|^2\rangle$ reflects a transition to the new scattering mode at the LHC energies and is valid for various U-matrix parameterizations which provide increasing total cross-sections. One should note that the numerical values of $\langle|S|^2\rangle$ at the Tevatron energies are in qualitative agreement with the number of quantitative calculations performed in the eikonal approaches (cf. [4]).

In the sense of the gap survival probability the situation should be more favorable at LHC energies since the obtained numerical values of $\langle|S|^2\rangle$ at these energies are close to unity, and this should lead to much higher cross-sections (by a factor of 40 compared to the calculations based on the gap survival probability estimations in the framework of the eikonal model [26]) e.g. for Higgs production in double diffractive processes compared to the values obtained with eikonal based estimations of the gap survival probability.

Thus, the antishadowing of which the appearance is expected at the LHC energies should be correlated with the enhancement of the Higgs production cross-section in double diffractive scattering and this would significantly help in detecting of the Higgs boson.

References

1. Y. Dokshitzer, V. Khoze, S. Troyan, in Physics in Collision VI, Proceedings of the International Conference, Chicago, Illinois, 1986, edited by M. Derrick (World Scientific, Singapore 1987), p. 365
2. A. Bialas, P.V. Landshoff, Phys. Lett. B **256**, 540 (1991)
3. J.D. Bjorken, Phys. Rev. D **45**, 4077 (1992); Phys. Rev. D **47**, 101 (1993)
4. V.A. Khoze, A.D. Martin, M.G. Ryskin, Eur. Phys. J. C **26**, 229 (2002)
5. R.S. Fletcher, Phys. Lett. B **320**, 373 (1994)
6. E. Gotsman, E. Levin, U. Maor, Phys. Lett. B **438**, 229 (1998)
7. M.M. Block, F. Halzen, Phys. Rev. D **63**, 114004 (2001)
8. A.B. Kaidalov, V.A. Khoze, A.D. Martin, M.G. Ryskin, Eur. Phys. J. C **21**, 521 (2001)
9. A. De Roeck, V.A. Khoze, A.D. Martin, R. Orava, M.G. Ryskin, Eur. Phys. J. C **25**, 391 (2002)
10. V.A. Petrov, R.A. Ryutin, hep-ph/0311024
11. R. Blankenbecler, M.L. Goldberger, Phys. Rev. **126**, 766 (1962)
12. A.A. Logunov, V.I. Savrin, N.E. Tyurin, O.A. Khrustalev, Theor. Math. Phys. **6**, 157 (1971)
13. W. Heitler, Proc. Cambr. Phil. Soc. **37**, 291 (1941)
14. S.M. Troshin, N.E. Tyurin, Phys. Lett. B **316**, 175 (1993)
15. S.M. Troshin, N.E. Tyurin, Eur. Phys. J. C **21**, 679 (2001); V.A. Petrov, A.V. Prokudin, S.M. Troshin, N.E. Tyurin, J. Phys. G **27**, 2225 (2001)
16. P. Giromini, Proceedings of the Vth Blois Workshop on Elastic and Diffractive Scattering, Providence, Rhode Island, June 1993, p. 30
17. T.T. Chou, C.N. Yang, Int. J. Mod. Phys. A **2**, 1727 (1987)
18. N.E. Tyurin, O.A. Khrustalev, Theor. Math. Phys. **24**, 847 (1976)
19. V.F. Edneral, S.M. Troshin, N.E. Tyurin, O.A. Khrustalev, Pisma Zh. Eksp. Teor. Fiz. **22**, 347 (1975); CERN-TH-2126 (1976)
20. P. Desgrolard, L. Jenkovszky, B. Struminsky, Eur. Phys. J. C **11** , 145 (1999)
21. S.M. Troshin, N.E. Tyurin, Nuovo Cim. A 106, 327 (1993); Proceedings of the Vth Blois Workshop on Elastic and Diffractive Scattering, Providence, Rhode Island, June 1993, p. 387; Phys. Rev. D **49**, 4427 (1994); Z. Phys. C **64**, 311 (1994)
22. S.M. Troshin, N.E. Tyurin, O.P. Yuschenko, Nuovo Cimento A **91**, 23 (1986); P.M. Nadolsky, S.M. Troshin, N.E. Tyurin, Z. Phys. C **69**, 131 (1995)
23. S.M. Troshin, N.E. Tyurin, Phys. Rev. D **55**, 7305 (1997)
24. S.M. Troshin, N.E. Tyurin, J. Phys. G **29**, 1061 (2003)
25. J. Bartels, H. Kowalski, Eur. Phys. J. C 693 (2001)
26. V.A. Khoze, A.D. Martin, M.G. Ryskin, Eur. Phys. J. C **18**, 167 (2000)